Modern University for Technology and Information
Faculty of Engineering
Mathematics (3) Final Examination
Fall season 2013, Time allowed 2 hours

Answer Only Five Questions:

(1- a) Test the following series for convergence

$$(i)\sum_{n=1}^{\infty}\frac{1}{n^2+1} \qquad (ii)\sum_{n=1}^{\infty}\frac{n}{3^n} \qquad (iii)\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$$

(1-b) Use the ratio test to find the interval of convergence of the series $\sum_{n=1}^{\infty}\frac{x^n}{2^n(n+1)}$.

---------- **(8 marks)**

(2) Solve the following differential equation

(a) $y' + y\tan x = \sin x$

(b) $xydx + \sqrt{1+x^2}dy = 0$

---------- **(8 marks)**

(3) Find the general solution of the following differential equation

(a) (a) $(D^2 - 2D + 1)y = \cos 3x$

(b) $(D^2 - 5D + 6)y = e^x \sinh 6x$

---------- **(8 marks)**

(4-a) Find $\nabla.\vec{F}$ *and* $\nabla\times\vec{F}$ given that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$

(4-b) Evaluate $\oint_C \vec{F}.d\vec{r}$ where $\vec{F} = (2x - 3y)\vec{i} + y\vec{j}$ and C is the circle $x^2 + y^2 = 4$

---------- **(8 marks)**

(5- a) Evaluate $\oint_C \frac{z^3 - 3z}{(z-2)}dz$ where C is the circle $|z| = 4$ in the complex plane.

(5- b) Evaluate $\int_{(3.0)}^{(0,3)} (2\bar{z} - z^2)dz$ on the circle $|z| = 3$.

(5 -c) Show that the function $f(z) = \sin z$ Satisfy Cauchy Remman's Equation

---------- **(8 marks)**

8 marks for each question Dr.Fathi Abdessalam